ECMM725 The Climate System

Example sheet 3

Please attempt all questions and hand in your work via the BART system (see the Education Office, Harrison Building) by 12.00 noon, Wednesday 24th January 2018.

Note: if you hand this assignment in late, a mark of 0 will be recorded, unless you make a case for leniency which is accepted by the Mitigation Committee.

Marks shown in questions are merely a guideline.

You may assume that the governing equations for incompressible flow in a rotating frame are

$$
\begin{aligned}
\rho\left(\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} + \mathbf{f}\times\mathbf{u}\right) &= -\nabla p + \rho\mathbf{g} + \mu\nabla^2\mathbf{u} \\
\nabla\cdot\mathbf{u} &= 0 \\
\frac{\mathrm{D}\rho}{\mathrm{D}t} &= 0
\end{aligned}
$$

1. Barotropic instability: Consider flow on the region $y \in [-L, L]$ which lies entirely within the Northern hemisphere. (Axes have been chosen such that x points east, y points north and $y = 0$ lies at the centre of the domain and **not** at the equator). Consider the following two velocity profiles of zonal flow:

$$
\begin{aligned}
U_1(y) &= U_0(y/L)^3 \\
U_2(y) &= U_0\frac{\exp(\lambda y)}{1+\exp(\lambda y)}
\end{aligned}
$$

where U_0 and λ are positive constants. By referring to Rayleigh's Theorem and Fjortoft's theorem, discuss whether barotropic instability is possible for each of these flows. Does it make a difference to your answer whether you consider the flow to be on a β–plane or an f–plane?

[15 marks]

2. Baroclinic instability: Give a full account of the Eady model of baroclinic instability for inviscid flow on an f–plane with constant buoyancy frequency N. (That is, starting from the quasi–geostrophic potential vorticity equation, go through the details of the derivation that is sketched in the lecture notes.)

[15 marks]

3. Wind–driven ocean circulation: Consider the Atlantic Ocean to be a rectangular basin, centred on 35°N, of longitudinal width $\mathrm{L_x}$ = 5000km and latitudinal width $\mathrm{L_y}$ = 3000km. The ocean is subjected to a zonal wind stress of the form

$$
\begin{aligned}
\tau_x(y) &= \tau_0\cos\frac{\pi y}{L_y} \\
\tau_y(y) &= 0
\end{aligned}
$$

where $\tau_0 = 0.1$Pa. Assume a constant value of $\beta = \mathrm{d}f/\mathrm{d}y$ appropriate to 35°N, and that the ocean has uniform density $\rho = 1000$kg.m^{-3}.

- Derive the Sverdrup relation, and hence determine the magnitude and spatial distribution of the depth-integrated meridional flow velocity in the interior of the ocean.
- Using the depth-integrated continuity equation, and assuming no flow at the eastern boundary of the ocean, determine the magnitude and spatial distribution of the depth-integrated zonal flow in the interior.
- If the return flow at the western boundary is confined to a width of 100km, determine the depth-integrated flow in this boundary current.
- If the flow is confined to the top 500m of the ocean (and is uniform with depth in this layer), determine the northward components of flow velocity in the interior, and in the western boundary current.
- Compute and sketch the pattern of Ekman pumping implied by the idealized wind pattern given above.

[20 marks]