ECMM725 The Climate System

# Example sheet 1

**Please attempt all questions and hand in your work via the BART system (see the Student Services Office, Harrison Building) by 12.00 noon, Wednesday 15th November 2017.**

Note: if you hand this assignment in late, a mark of 0 will be recorded, unless you make a case for leniency which is accepted by the Mitigation Committee.

Marks shown in questions are merely a guideline.

Final marks will be expressed as perecentages.

1. **Dimensional analysis:**

   Show from first principles (i.e. using formulae from 'school physics' where necessary) that the scale height of the atmosphere $RT_0/g$ has units of length, and that the dry adiabatic lapse rate $g/c_p$ has units of absolute temperature per unit length.

   **[5 marks]**

2. **Planetary energy balance:**

   Consider the following data: The solar constant (radiative flux reaching top of earth's atmosphere) is $S_0 = 1370$ W.m$^{-2}$. Data for other planets are

| Planet | dist. to sun (AU) | $T_0$ (K) | $p_0$ (bar) | $g$ (m.s$^{-2}$) | $R$ (J.kg$^{-1}$.K$^{-1}$) |
|---|---|---|---|---|---|
| Earth | 1 | 280 | 1 | 9.81 | 287 |
| Venus | 0.72 | 750 | 90 | 8.84 | 188 |
| Mars | 1.52 | 218 | 0.006 | 3.76 | 188 |
| Jupiter | 5.20 | 134 | 2 | 26 | 4133 |

   If these planets had no atmospheres, and all had a uniform albedo $a = 0.05$ what would be their surface temperatures?

   **[5 marks]**

3. **Geopotential thickness:**

   Estimate typical values for the geopotential thickness of the $1000 - 500$ hPa layer

   (a) when the surface temperature is 0°C

   (b) when the surface temperature is 20°C.

   By how much would the geopotential thickness of the $1000 - 500$ hPa layer increase under uniform heating by 1°C

   **[5 marks]**

4. **Adiabatic compression:**

Concorde used to fly at a pressure of around 100 hPa, where the typical air temperature is around $-50$ °C. If the cabin pressure in Concorde is around 850 hPa, what would be the consequences of adiabatic compression of the outside air to provide ventilation?

**[5 marks]**

5. **Buoyancy frequency:**

Show that, for an isothermal atmosphere, the square of the Brunt-Väisäla frequency is given by

$$N^2 = \frac{g^2}{c_p T_0}$$

The isothermal atmosphere is one of the most stable profiles observed in the atmosphere (for example, in the lower stratosphere), and so this provides an upper bound for the buoyancy frequency. Hence show that typical buoyancy periods are longer than $\sim 5$ minutes.

**[10 marks]**

6. **Diurnal radiative cycle:**

Consider the diurnal (daily) cycle of shortwave radiation absorbed at a fixed point on the earth's surface. Let $\tau$ (=24 hours = 86,400 seconds) be the length of the day/night cycle, and let $d$ be the time between sunrise and sunset. Set $t = 0$ to be sunrise, and assume that the diurnal cycle of absorbed shortwave radiation $I(t)$ (W.m$^{-2}$) is given by:

$$I(t) = \begin{cases} I_0 \sin \frac{\pi t}{d} & t \in [0, d] \\ 0 & t \in [d, \tau]. \end{cases}$$

In this question we will consider a simple model for changes in surface temperature $T(t)$ during the day and night.

(a) Suppose that the sun lies at an angle $\pi/2 - \phi$ above the horizon at noon. Give an expression relating $I_0$ to the solar constant $S = 1370$ W.m$^{-2}$, albedo $a$ and angle $\phi$. Give a sketch graph to describe qualitatively how $d$ and $\phi$ vary throughout the year.

(b) Show that the daily–mean absorbed shortwave radiation is

$$\bar{I} = \left( \frac{2d}{\pi \tau} \right) I_0.$$

(c) By referring to standard formulae, explain why

$$\bar{T} = \left( \frac{\bar{I}}{\epsilon \sigma} \right)^{1/4}$$

is a reasonable definition of 'typical' surface temperature.

(d) Explaining clearly any approximations you make, show that

$$O(t) = \epsilon\sigma\bar{T}^4 + 4\epsilon\sigma\bar{T}^3(T(t) - \bar{T})$$

is a reasonable expression for the outgoing longwave radiation from the surface.

(e) Assuming that the diurnal temperature cycle is periodic (i.e. that there is no difference between subsequent days), show that the daily–mean outgoing radiation is

$$\bar{O} = \epsilon\sigma\bar{T}^4.$$

(f) Suppose that the surface temperature evolves according to the governing equation

$$c\frac{dT}{dt} = I(t) - O(t)$$

where $c = 4 \times 10^6 \mathrm{J.m^{-2}.K^{-1}}$ is an effective heat capacity per unit area.
Explain how the absorbed shortwave radiation can be rewritten in complex form as

$$I(t) = \begin{cases} -iI_0 \exp\frac{i\pi t}{d} & t \in [0, d] \\ 0 & t \in [d, \tau] \end{cases}$$

where $i = \sqrt{-1}$. (By convention, it is assumed that only the real part of this expression has physical meaning.)

(g) Define the constant $B = 4\epsilon\sigma\bar{T}^3$.
By reference to the governing equation, give a physical interpretation of the quantity $c/B$.
Show that a solution of the form

$$T(t) = \begin{cases} \left(a_0 + a_1 \exp\frac{-Bt}{c} + a_2 \exp\frac{i\pi t}{d}\right)\frac{I_0}{B} & t \in [0, d] \\ \\ \left(a_3 + a_4 \exp\frac{-Bt}{c}\right)\frac{I_0}{B} & t \in [d, \tau] \end{cases}$$

satisfies the governing equation for suitable (possibly complex) constants $\{a_0, \ldots, a_4\}$. Calculate expressions for these constants which are functions of dimensionless ratios (such as $Bd/c$ and $B\tau/c$).

(h) Use suitable software (e.g. R or Matlab) to plot $T(t)$ for $t \in [0, \tau]$ for reasonable values of the input parameters. Hence, and with reference to the governing equation, discuss why it is that the coldest part of the day occurs **slightly after** sunrise. Give an estimate for the time lag between sunrise and the coldest part of the day.

**[20 marks]**

7. **Moist thermodynamics and tephigrams:**

Consider the following radiosonde data, acquired at midnight local time:

| $p$ (kPa) | $T$ (°C) | $T_d$ (°C) |
|---|---|---|
| 100 | 13 | 11 |
| 94 | 9.5 | 8 |
| 90 | 7 | 5 |
| 78 | 0 | -3 |
| 70 | -5 | -11 |
| 60 | -11 | -17 |
| 50 | -20 | -28 |
| 40 | -32 | -42 |
| 30 | -47 | - |
| 20 | -49 | - |
| 15 | -50 | - |
| 10 | -48 | - |

Plot these profiles on a tephigram chart (which you should include) and answer the following questions, referring where necessary to the tephigram:

(a) what is the pressure at the tropopause?

(b) what parts of the ascent are stable for (a) dry air, and (b) for saturated air?

(c) what is the humidity (a) at 100 kPa, and (b) at 50 kPa.

(d) If air at the surface is heated during the following day and then rises adiabatically, at what level will condensation occur?

(e) What will be the level of the top of any convective clouds which form during the day?

(f) Suppose that the column of air in the radiosonde data is forced to ascend over a range of mountains. Consider the layer of air initially between 78kPa and 70kPa. Show that is stable for both dry and saturated ascent. Suppose the layer is forced to rise by 10kPa in pressure units. Explain why the layer will remain 8kPa deep in pressure units. Find the new temperatures of the bottom and top of the layer, by taking them up the dry adiabat until they become saturated, and then up the moist adiabat. Show that the layer is now unstable with respect to saturated ascent.

(g) Take air from the surface up the dry adiabat to the lifting condensation level (Normand's point), and then up the moist adiabat until to the level of free convection, and then the level of neutral buoyancy. Estimate the convective inhibition (CIN) and convective available potential energy (CAPE) in this case.

**[20 marks]**

8. **Temperature inversion in the stratosphere:**

Consider a model for plane–parallel atmospheric radiation in which longwave (infra–red) radiation is represented by $L\uparrow$ and $L\downarrow$, and in which shortwave radiation is split into two bands: ultraviolet ($U\uparrow$ and $U\downarrow$) and visible ($V\uparrow$ and $V\downarrow$). Let $B$ denote longwave blackbody emission by the atmosphere, and let $x$ be LW optical height in the atmosphere. You may assume that:

$$
\begin{array}{rcll}
\alpha & = & 0.3 & \text{albedo of surface} \\
\beta & = & 0.5 & \text{visible fraction of incoming SW} \\
\rho & = & \rho_0 \exp(-z/H) & \text{density as function of height, where} \\
H & = & 10\text{km} & \text{scale height of atmosphere} \\
S_0 & = & 400\text{W.m}^{-2} & \text{total incoming SW radiation at TOA} \\
x_0 & = & 3 & \text{optical thickness of atmosphere to LW (due to CO}_2\text{)} \\
y_0 & = & 6 & \text{optical thickness of atmosphere to UV (due to O}_2\text{)}
\end{array}
$$

For the purposes of this question you may further assume that the extinction coefficients for each band are independent of height.

(a) Show from the definition of optical depth that

$$x = x_0 \, (1 - \exp(-z/H))$$

(b) Assuming that visible light is not attenuated by the atmosphere, justify the following governing equations:

$$
\begin{aligned}
\frac{\mathrm{d}U\downarrow}{\mathrm{d}x} &= \frac{y_0}{x_0} U\downarrow \\
\frac{\mathrm{d}U\uparrow}{\mathrm{d}x} &= -\frac{y_0}{x_0} U\uparrow \\
\frac{\mathrm{d}V\downarrow}{\mathrm{d}x} &= 0 \\
\frac{\mathrm{d}V\uparrow}{\mathrm{d}x} &= 0 \\
\frac{\mathrm{d}L\downarrow}{\mathrm{d}x} &= -B + L\downarrow \\
\frac{\mathrm{d}L\uparrow}{\mathrm{d}x} &= B - L\uparrow
\end{aligned}
$$

(c) Define $S\uparrow = U\uparrow + V\uparrow$ and $S\downarrow = U\downarrow + V\downarrow$. Use the governing equations and suitable boundary conditions (which you should state) to show that the shortwave fluxes must be:

$$
\begin{aligned}
S\downarrow &= \beta S_0 + ((1-\beta) S_0 \exp(-y_0)) \exp(y_0 x / x_0) \\
S\uparrow &= \alpha\beta S_0 + (\alpha(1-\beta) S_0 \exp(-y_0)) \exp(-y_0 x / x_0)
\end{aligned}
$$

(d) Define net fluxes $S_n = S\uparrow - S\downarrow$ and $L_n = L\uparrow - L\downarrow$. Explain physically why

$$L_n + S_n = 0$$

for all $x$ and hence show that

$$2B - L\downarrow - L\uparrow + \frac{\mathrm{d}S_n}{\mathrm{d}x} = 0$$

(e) Deduce explicit functional forms for $L\downarrow(x)$, $L\uparrow(x)$ and $B(x)$. (*Hint*: try a solution of the form $L\downarrow = a + bx + c\exp(y_0 x/x_0) + d\exp(-y_0 x/x_0)$, and apply a suitable boundary condition at $x = x_0$.)

(f) Use suitable software (e.g. R or Matlab) to plot vertical profiles for the long-wave, shortwave and blackbody fluxes as functions of geometric height $z$.

(g) Use these and similar plots to justify the statements **There is a temperature inversion in the stratosphere** and **Under climate change, the troposhere warms while the stratosphere cools**. (*Hint*: consider how $x_0$ and $y_0$ would change under elevated levels of $CO_2$.)

**[20 marks]**