ECMM723 – Modelling weather and climate
Problem sheet 1

January 30, 2018

Please attempt questions marked ⋆ and hand in your work via the BART system (see Student Services, Harrison Building) by 12.00 noon, Monday 26th February 2018. Questions marked *(optional)* are not compulsory and carry no mark.

1. ⋆ The Clausius-Clapeyron equation for the saturation vapour pressure of water may be written

$$\frac{d \ln p_w}{dT} = \frac{L}{RT^2},$$

where p_w is saturation vapour pressure, T is temperature, L is the latent heat of vaporisation and R is the specific gas constant. Calculate the height at which the water vapour pressure is reduced to $\frac{1}{e}\times$ its surface value for an atmosphere with a surface temperature $T_0 = 298$ K and a constant lapse rate $\Gamma = -\frac{dT}{dz} = 7 \times 10^{-3}$ Km^{-1}. This is known as the water vapour pressure scale height. You may assume that the atmosphere is saturated and that L is a constant. **[8]**

2. ⋆ Consider a planet on which atmospheric optical depth, χ, decreases with some absorber scale height, H, so that

$$\chi = \chi_0 e^{-\frac{z}{H}},$$

where χ_0 is total optical depth measured at the surface.

(a) If we assume that the tropopause occurs at $\chi = 1$, show that tropopause height, z_{trop}, is given by

$$z_{trop} = H \ln \chi_0.$$

[4]

(b) Hence find a numerical estimate for the tropopause height for Earth, given that $\chi_0 \sim 5$ and $H \sim 4$ km. **[2]**

(c) Find a numerical estimate for the absorber scale height, H, on Venus, assuming that the atmosphere is composed entirely of CO_2, and is in hydrostatic equilbrium. You may assume that the atmosphere is isothermal, with temperature $T_0 = 500$ K. **[8]**

(d) Hence find a numerical estimate for the tropopause height for Venus, given that $\chi_0 \sim 100$. **[2]**

3. ⋆ A zero dimensional energy balance model of a planet's global mean surface temperature T as a function of time, t, that depends only on insolation, S, and the partial pressure of atmospheric CO_2, p_{CO2}, can be written

$$c\frac{dT}{dt} = \frac{S}{4}(1-A) - \frac{S_0}{4}(1-A_w) - \lambda(T-T_0) + b\ln\left(\frac{p_{CO2}}{p_0}\right).$$

A is the planet's albedo, which depends on T. A_c is the albedo in a cold ice-covered state. A_w is the albedo in a warm ice-free state. $0 < A_w < A_c < 1$. The switch between states happens suddenly at T_0 such that:

$$T < T_0: \qquad A = A_c,$$
$$T \geq T_0: \qquad A = A_w.$$

c, λ, T_0, b, S_0 and p_0 are positive constants.

On geological timescales p_{CO2} is controlled by the balance between volcanic emission, V, and loss due to weathering[1], such that

$$\frac{dp_{CO2}}{dt} = V - W_0 e^{k(T-T_0)}\left(\frac{p_{CO2}}{p_0}\right)^{\beta}.$$

W_0, β and k are positive constants.

(a) In steady state, show that $T - T_0$ may be written

$$T - T_0 = \frac{b\ln\left(\frac{V}{W_0}\right) + \beta\left(\frac{S}{4}(1-A) - \frac{S_0}{4}(1-A_w)\right)}{kb + \lambda\beta}.$$

[5]

(b) Consider the upper limit of the cold state and the lower limit of the warm state. For a given value of S, where $S > S_0$, find the domain of $\ln\left(\frac{p_{CO2}}{p_0}\right)$ over which both the warm and cold steady states can exist in terms of S and the constants. **[7]**

(c) With reference to the equations, explain qualitatively how and why volcanism must differ between the warm and cold states for the same value of p_{CO2} in part (b). **[2]**

[1] CO_2 dissolved in rainwater reacts with rocks, removing CO_2 from the atmosphere.

Another planet has constant volcanism $V = V_0$, and constant weathering, W_0, that only operates in the warm state[2] so that

$$T < T_0 : \qquad \frac{dp_{CO2}}{dt} = V_0,$$
$$T \geq T_0 : \qquad \frac{dp_{CO2}}{dt} = V_0 - W_0.$$

$W_0 > V_0 > 0$.

(d) For constant $S = S_1$, where $S_1 > S_0$, sketch the time evolution of p_{CO2} and T, given that the planet starts in the cold state and $p_{CO2}(t = 0) = p_0$. You may assume that $c\frac{dT}{dt}$ is small and $\frac{S_1}{4}(1 - A_c) < \frac{S_0}{4}(1 - A_w)$. Your sketches should be qualitative, but show the functional form of the curves, indicate the position of any discontinuities and show the position of turning points of T and p_{CO2} relative to their intial states. **[10]**

4. $\star$ The heat transport of the Stommel box model of the thermohaline circulation from Topic 4, Section 3 is due to the upper branch of q and is therefore $\rho C_p q T'$, where C_p is the specific heat capacity of sea water, ρ is density, q is flow rate and T' is the temperature difference between the equatorial and polar boxes. (We don't need to consider the heat transport due to the lower branch of q because its heat flow is balanced by solar heating in the equatorial box and cooling to the atmosphere in the polar box.)

Using the constants from Topic 4, Figure 9, and a computer where necessary:

(a) Plot or sketch the heat transport due to q_1 in the range -1 Sv $< E <$ +1 Sv. **[3]**

(b) Assume that q_1 is the only heat transport and that $E < 0.5$ Sv always. If E suddenly increases, what is the initial effect on q_1 before T' changes? Without doing detailed calculations, describe the subsequent response of T' and q_1 to the perturbation. Is the system stable or unstable? Why? (Here "Stable" means that a small perturbation to the system is opposed by net negative feedbacks, that attempt to return the system to the previous equilibrium. "Unstable" means that the perturbation grows, due to net positive feedbacks, and the system is driven further away from its original equilibrium.) **[5]**

(c) Write down an expression for the heat transport due to the freshwater term, E. (*Hint: E works by evaporating water from the equatorial box and condensing it in the polar box.*) **[3]**

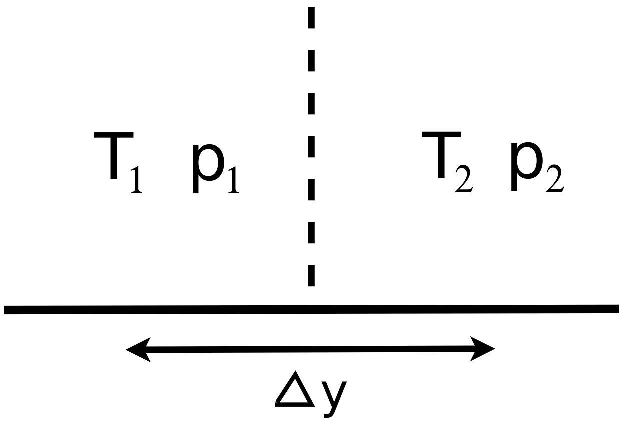
(d) Overplot the heat transport due to E on your diagram. **[2]**

(e) Does the heat transport due to E tend to make the whole system more or less stable? **[1]**

[2] The idea is that if a planet is ice-bound, then weathering is probably greatly reduced.

Optional / revision questions

5. *(optional)*



An atmosphere consists of dry air, has temperature as a function of horizontal coordinates x and y but not vertical coordinate z, is in hydrostatic balance in the vertical and in geostrophic balance in the horizontal (ie $-fu = \frac{1}{\rho}\frac{\partial p}{\partial y}$). In this case, vertical variation of the x component of velocity, u, is given by thermal wind balance

$$f\frac{\partial u}{\partial z} = -\frac{g}{T}\frac{\partial T}{\partial y}.$$

Consider two air masses lying side-by-side with surface pressures $p_{1,S}$ and $p_{2,S}$ and vertically uniform temperatures T_1 and T_2.

(a) Find expressions for the pressures p_1 and p_2 as a function of z in each column.

(b) Where $p_{1,S} > p_{2,S}$ and $T_1 < T_2$, sketch p_1 and p_2 against z on one set of axes.

(c) Hence, find the height where $u = 0$, when $p_{1,S} = 1000$ hPa, $p_{2,S} = 985$ hPa, $T_1 = 275$ K and $T_2 = 285$ K.

(d) A warm surface low pressure is observed to sit next to a cold high surface pressure in the Northern Hemisphere. The systems are deep enough such that a layer of no motion ($u = 0$) exists. Based on your answers to the earlier parts, describe and explain the behaviour of pressure and winds that you would expect to see below and above the layer of no motion.

6. *(optional)*

   Consider a marginal sea with inflow and outflow occurring at a rate q. Inflow has no temperature or salinity anomaly with respect to some arbitrary baseline i.e. inflow has $T' = S' = 0$. It is assumed that the sea is well-mixed, hence outflow has the mean sea temperature anomaly T' and salinity anomaly S'. The equations of state for temperature and salinity are

$$\frac{dT}{dt} = c(T_0 - T') - qT',$$
$$\frac{dS}{dt} = d(S_0 - S') - qS'.$$

   (a) Using $\delta = \frac{d}{c}$, $f' = \frac{q}{c}$, $x = \frac{S'}{S_0}$ and $y = \frac{T'}{T_0}$, show that in steady-state these equations can be written "non-dimensionally" as

$$1 - (1 + f')y = 0,$$
$$\delta - (\delta + f')x = 0.$$

   Eliminate f' and find the relationship between x and y.

   (b) The perturbation equation of state (ie for small changes with respect to some background) for sea water may be written $\rho = \rho_0(1 - \alpha T' + \beta S')$, where α, β and ρ_0 are positive constants. Using the non-dimensional quantities above, show that

$$\rho = \rho_0(1 + \alpha T_0(-y + Rx)),$$

   where $R = \frac{\beta S_0}{\alpha T_0}$.

   (c) In the domain $x : 0 \to 1$,

   - Sketch y against x for $\delta = 1$.
   - And for some value of $\delta < 1$.
   - Overplot lines of constant ρ for some positive value of R. (ie lines of $-y + Rx =$ *const.*).

   (This may be best done using a computer.)

   What is the impact on outflow density as f' increases from 0 to ∞ for $\delta = 1$ and $\delta < 1$? What does this mean physically?

7. *(optional)* A horizontal slab of atmosphere of longwave optical depth $\Delta\chi$ and temperature T_{slab} is present in an atmosphere that is otherwise at temperature T_{atm}.

(a) By integrating either the upward or the downward differential equation for longwave radiative transfer and using symmetry or otherwise, show that the net heat lost by the slab due to longwave radiation is

$$q_{lost} \simeq 2\sigma\Delta\chi(T_{slab}^4 - T_{atm}^4),$$

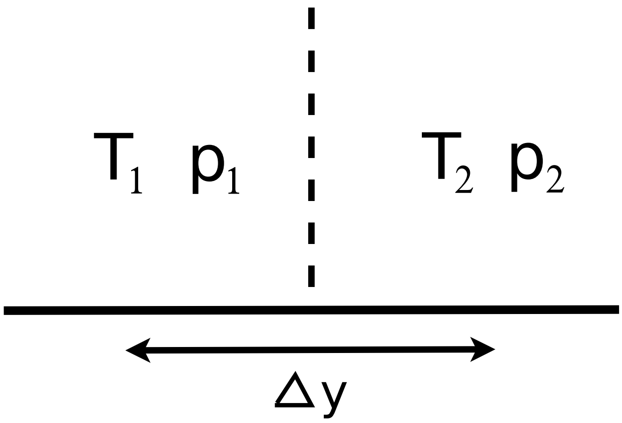
where the approximation $e^{-x} \simeq 1-x$ has been employed. You may assume that radiative transfer is "grey" (ie absorption is constant across the spectrum and $\pi B = \sigma T^4$) and that radiation due to the atmosphere surrounding the slab is blackbody ($F = \sigma T_{atm}^4$).

(b) By making the approximation $(T_{slab}^4 - T_{atm}^4) \sim 4T_{atm}^3\Delta T$ where $\Delta T = T_{slab} - T_{atm}$, and by assuming that T_{atm} is a constant, find an expression for the rate of change of the temperature of the slab due to q_{lost} given that the slab has heat capacity c.

(c) Find the time it takes for $\Delta T \to \frac{\Delta T}{e}$ when $T_{atm} = 270$ K, $c = 9 \times 10^5$ JK^{-1}m^{-2} and $\Delta\chi = 0.2$. This is known as the radiative relaxation timescale.

(d) What does the radiative relaxation timescale tell you about the role of radiation in the development of meteorological phenomena?

8. *(optional)*



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