Modelling the weather and climate

Coursework 2: Numerical methods and sub-grid processes

Please hand in your work to the Education Office by 12 noon, Thursday 28 March, 2018. Note: if you hand this assignment in late a mark of 0 will be recorded, unless you make a case for leniency that is accepted by the Mitigation Committee.

Please hand in your workings and MATLAB figures for the questions below. Also hand in fully-commented versions of the MATLAB codes written.

1. **Numerical methods**

Consider the linear advection equation

$$\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} = 0,$$

on a periodic domain $0 \leq x \leq 1$. Consider also the two different initial conditions:

$$\phi(x,0) = \begin{cases} \frac{1}{2}\{1+\cos[4\pi(x-1/2)]\} & \text{if } 1/4 \leq x \leq 3/4 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$\phi(x,0) = \begin{cases} 1 & \text{if } 1/4 \leq x \leq 3/4 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

(a) Write a MATLAB code to integrate the linear advection equation until $t = 0.5$ using a centred-in-time, centred-in-space (CTCS) discretization. Perform the integration for both sets of initial conditions. Take $u = 2.0$, use $N = 51$ equally spaced gridpoints, and take $\Delta t = 0.005$. You might find it helpful to follow the structure of `diffuse.m` (available on the ele module web-page) and use the same tricks for coping with the periodic boundary condition. (30)

(b) Discretise the advection equation using a semi-Lagrangian scheme, and cubic-Lagrange interpolation. The departure points can be evaluated using a simple linear formula, i.e. a displacement of $u\Delta t$ upstream. Demonstrate how this formulation improves on the CTCS scheme when increasing the time step above the CFL limit. (35)

2. **Sub-grid processes**

Holton, section 5.3 gives a mixed-layer model of the atmospheric boundary layer

$$\overline{u} = \overline{u_g} - \kappa_s|\overline{\mathbf{V}}|\overline{v}, \qquad \overline{v} = \kappa_s|\overline{\mathbf{V}}|\overline{u},$$

where the turbulence magnitude is given by $\kappa_s = C_d/(fh)$.

(a) Explain the relationship between the above mixed-layer equations and the Reynolds averaged momentum equations, explaining carefully the meaning of the notation. (3)

(b) You are given the code `mixed_layer_wind1.m`. Modify the code to calculate the wind turning (the angle between the geostrophic and mixed layer winds) for a range of values of h. Use the values: u_g =10 ms^{-1} C_d =2 $\times$ 10^{-3} and f =10^{-4} s^{-1}. Calculate and plot the wind turning for values of h in the range of 300 m to 3000 m. (12)

(c) You are given the code `mixed_layer_wind2.m` which prescribes the geostrophic wind (u_g, v_g) at each horizontal point. Calculate the mixed-layer wind ($\overline{u}$, $\overline{v}$) at each horizontal point using the algorithm in `mixed_layer_wind1.m`. *Hint: you may want to incorporate the mixed-layer algorithm as a function.* Plot the mixed-layer wind as a vector-wind plot (as done for the geostrophic wind). Assuming a constant boundary-layer depth of 1000 m, calculate the Ekman pumping for each horizontal point and plot the results. (20)

[**100**]